Collapse Universality in the UNNS Substrate
From Class-Level Uniqueness to Quantitative Stability

UNNS Research Collective

Abstract

We formalize collapse universality in the Unbounded Nested Number Sequences (UNNS) Substrate as a pre-regularity principle governing structural survival. First, we establish a qualitative universality theorem: Operator XII maps all supercritical structures within the same coarse descriptor class to a unique seed class, independent of microscopic detail. We then extend this result to a quantitative stability theorem by endowing the descriptor space with a metric, proving that collapse induces a Lipschitz (and potentially contractive) map on universality labels. This progression clarifies UNNS as a pre-regularity theory: stability and universality are properties of collapse on structural descriptors, preceding geometry, smoothness, and PDE-based regularity.

1 Autonomy and Scope

UNNS dynamics is autonomous. It assumes:

- no continuum domain,
- no differentiable structure,
- no partial differential equations.

Collapse universality is therefore not a statement about solutions evolving in time, but about structural survival under admissibility constraints.

2 Structures and Nesting

Definition 1 (Structure Space). *Let $(\mathcal{S}, \preceq)$ be a partially ordered set of structures, where $S' \preceq S$ denotes "S' is a substructure of S".*

Definition 2 (Nesting Operator). *A nesting operator is a map*

$$\mathcal{N} : \mathcal{S} \to \mathcal{S}^{\mathbb{N}}, \qquad \mathcal{N}(S) = (S_0, S_1, S_2, \dots),$$

with $S_{n+1} \preceq S_n$ for all n.

3 Admissibility and τ-Energy

Definition 3 (Feature Space and Admissibility). *Let $(\mathcal{F}, d)$ be a metric space, $F : \mathcal{S} \to \mathcal{F}$ a feature extractor, and $\mathcal{A} \subseteq \mathcal{F}$ the admissible feature set.*

Definition 4 (Mismatch). *For $S \in \mathcal{S}$ define*

$$\Delta(S) := \mathrm{dist}(F(S), \mathcal{A}) = \inf_{a \in \mathcal{A}} d(F(S), a).$$

Definition 5 (τ-Energy Functional). *Let $w_n > 0$ with $\sum_{n \geq 0} w_n < \infty$. For $\mathcal{N}(S) = (S_n)$ define*

$$\mathcal{E}_\tau(S) := \sum_{n=0}^{\infty} w_n \, \Delta(S_n).$$

Fix $\tau_{\mathrm{crit}} > 0$. A structure is subcritical *if $\mathcal{E}_\tau(S) < \tau_{\mathrm{crit}}$.*

$\mathcal{E}_\tau$ is not an energy over space or fields; it is a *survivability budget* over recursive depth.

4 Operator XII (Collapse)

Definition 6 (Collapse Operator). *Operator XII is a map $C : \mathcal{S} \to \mathcal{S}$, interpreted as collapse $\to$ seed.*

Axiom 1 (Seed Output). *For all $S \in \mathcal{S}$,*

$$\mathcal{E}_\tau(C(S)) < \tau_{\mathrm{crit}}, \qquad C(C(S)) = C(S).$$

Collapse is terminal with respect to the pre-collapse structure and produces a τ-persistent seed.

5 Descriptor Space and Coarse-Graining

Definition 7 (Descriptor Map). *Let $\Pi : \mathcal{S} \to \mathcal{U}$ be a surjective map into a set $\mathcal{U}$ of universality descriptors.*

Axiom 2 (Descriptor-Respecting Collapse). *If $\Pi(S) = \Pi(T)$ then $\Pi(C(S)) = \Pi(C(T))$.*

This axiom formalizes micro-insensitivity: collapse depends only on descriptor-level information.

6 Qualitative Collapse Universality

Theorem 1 (Class-Level Universality). *There exists a unique map $\sigma : \mathcal{U} \to \mathcal{U}$ such that for all $S \in \mathcal{S}$,*

$$\sigma(\Pi(S)) = \Pi(C(S)).$$

Moreover, σ is idempotent: $\sigma(\sigma(u)) = \sigma(u)$.

Proof. Define $\sigma(u) = \Pi(C(S))$ for any S with $\Pi(S) = u$. Well-definedness follows from descriptor-respecting collapse. Idempotence follows from $C(C(S)) = C(S)$. $\square$

Interpretation

All supercritical structures within the same descriptor class collapse to the same seed class. Collapse universality is therefore qualitative and structural, not geometric or analytic.

7 Quantitative Extension: Metric Stability

We now enrich the descriptor space.

Definition 8 (Metric Descriptor Space)**.** *Assume* $(\mathcal{U}, d_{\mathcal{U}})$ *is a metric space.*

Axiom 3 (Quantitative Descriptor Stability)**.** *There exists* $L_C \geq 0$ *such that for all* $S, T \in \mathcal{S}$*,*

$$d_{\mathcal{U}}(\Pi(C(S)), \Pi(C(T))) \leq L_C \, d_{\mathcal{U}}(\Pi(S), \Pi(T)).$$

8 Quantitative Collapse Universality

Theorem 2 (Lipschitz Universality)**.** *The induced seed map* $\sigma : \mathcal{U} \to \mathcal{U}$ *is Lipschitz with constant* L_C*:*

$$d_{\mathcal{U}}(\sigma(u), \sigma(v)) \leq L_C \, d_{\mathcal{U}}(u, v).$$

Proof. Choose representatives S, T with $\Pi(S) = u$, $\Pi(T) = v$ and apply the stability axiom. $\square$

Corollary 1 (Quantitative Micro-Insensitivity)**.** *Small changes in descriptors before collapse produce proportionally small changes in seed descriptors after collapse.*

9 Contractive Regime

Definition 9 (Contractive Collapse)**.** *Collapse is contractive if* $L_C < 1$*.*

Theorem 3 (Attractor Property)**.** *If collapse is contractive, then* σ *has a unique fixed descriptor* $u^* \in \mathcal{U}$*, and for all* $u \in \mathcal{U}$*,*

$$d_{\mathcal{U}}(\sigma^k(u), u^*) \leq L_C^k \, d_{\mathcal{U}}(u, u^*).$$

This expresses universality as convergence toward a canonical seed class.

10 UNNS as Pre-Regularity Theory

The progression is now explicit:

- **Qualitative stage:** collapse defines canonical seed classes.
- **Quantitative stage:** collapse depends stably on descriptors.
- **Downstream only:** geometry, smoothness, and PDE regularity may appear as representations of persistent seeds.

Regularity is therefore not imposed by equations but emerges from survivability.

11 Conclusion

Collapse universality in UNNS has both qualitative and quantitative content. Operator XII induces a deterministic, stable projection on descriptor space, independent of microscopic detail and prior to geometric formulation. This places UNNS firmly as a pre-regularity theory: it explains why regularity principles exist, rather than reproducing them.

12 Chamber-Realized Mismatch and Observable τ-Energy

The abstract definition of mismatch $\Delta(S_n)$ admits concrete realizations via chamber-level observables. This section specifies an explicit construction used in UNNS chambers, establishing a direct link between τ-energy and measured quantities.

12.1 Observable Feature Vector

At nesting depth n, let the chamber compute an observable feature vector

$$F(S_n) = (\tau_n,\ \|\nabla\tau\|_n,\ \kappa_n,\ g_n,\ \dots),$$

where each component corresponds to a logged chamber observable: τ-range values, gradient magnitudes, curvature proxies, stability scores, or other admissibility-relevant quantities.

12.2 Admissibility Region

Define the admissible feature set as a Cartesian constraint region

$$\mathcal{A} = \left\{ f \in \mathcal{F} : \begin{array}{c} \tau_{\min} \le \tau \le \tau_{\max}, \\ \|\nabla\tau\| \le G_{\max}, \\ \kappa_{\min} \le \kappa \le \kappa_{\max}, \\ \dots \end{array} \right\}.$$

These bounds are chamber-defined and correspond exactly to admissibility filters enforced during simulation.

12.3 Penalty-Based Mismatch

Define mismatch as a weighted admissibility violation:

$$\begin{aligned} \Delta(S_n) := {} & \alpha_\tau\,[\tau_n - \tau_{\max}]_+ + \alpha_\tau\,[\tau_{\min} - \tau_n]_+ \\ & + \alpha_G\,[\|\nabla\tau\|_n - G_{\max}]_+ \\ & + \alpha_\kappa\,\mathrm{dist}(\kappa_n, [\kappa_{\min}, \kappa_{\max}]) + \cdots, \end{aligned}$$

where $[x]_+ = \max(x, 0)$ and $\alpha_i > 0$ are chamber-fixed weights.

Interpretation

With this construction, the τ-energy

$$\mathcal{E}_\tau(S) = \sum_n w_n \Delta(S_n)$$

is not abstract: it is a depth-weighted accumulation of measured admissibility violations. Chamber XXVIII computes a direct surrogate of $\mathcal{E}_\tau$ through its logged observables and filters.

13 Empirical Signature: Chamber XIV and Local Contraction

Chamber XIV demonstrates emergence of a preferred scale parameter $\mu^\star \approx 1.618$ through minimization of a scale-loss function $\Delta_{\text{scale}}(\mu)$.

13.1 Scale Descriptor Slice

Restrict the descriptor space to a one-dimensional scale slice

$$\mathcal{U}_\mu := \mathbb{R}_{>0}, \qquad d_{\mathcal{U}}(\mu_1, \mu_2) = |\mu_1 - \mu_2|.$$

Within this slice, define the collapse-induced selector

$$\sigma(\mu) := \arg\min_{\tilde{\mu} \in \Omega} \Delta_{\text{scale}}(\tilde{\mu}),$$

where Ω is the admissible search interval used by the chamber.

13.2 Local Attractor Interpretation

Empirically, $\Delta_{\text{scale}}(\mu)$ exhibits a convex minimum at $\mu^\star \approx \phi$. This implies:

- uniqueness of the seed descriptor $u^\star = \mu^\star$,
- stability of σ under small perturbations of μ,
- a locally contractive regime of collapse near $\mu^\star$.

Remark

This observation supports *local* contraction of the descriptor map σ near $u^\star$. It does not assert global contractivity across all descriptors. Chamber XIV therefore provides empirical evidence for a basin of attraction in descriptor space rather than a universal contraction law.

14 Regimes of Collapse and β-Coupling

UNNS operators may induce either stabilizing or dispersive behavior depending on coupling strength. In particular, Operator XV introduces a dispersive β-coupling between structural channels.

14.1 -Dependent Descriptor Stability

Let $\beta \geq 0$ parameterize dispersive coupling strength. Assume the descriptor-level collapse bound takes the form

$$d_{\mathcal{U}}\big(\Pi(C_\beta(S)), \Pi(C_\beta(T))\big) \leq L_C(\beta)\, d_{\mathcal{U}}\big(\Pi(S), \Pi(T)\big),$$

where $L_C(\beta)$ is monotone non-decreasing in β.

14.2 Three Regimes

This yields three dynamical regimes:

- **Contractive regime:** $L_C(\beta) < 1$. Collapse stabilizes descriptors and produces attractor seed classes.
- **Critical regime:** $L_C(\beta) = 1$. Descriptor differences are preserved.
- **Dispersive regime:** $L_C(\beta) > 1$. Collapse amplifies descriptor differences, enabling branching or diversification.

Interpretation

Dispersive coupling (XV) and contractive collapse (XII) are not contradictory. They correspond to distinct substrate regimes. Empirical chambers may probe local contractive basins (as in Chamber XIV) even when the global system admits dispersive phases.

Synthesis: Chambers as Empirical Probes of Pre-Regularity

Chambers XIV and XXVIII should be understood not as numerical solvers or downstream physical simulations, but as empirical probes of the UNNS pre-regularity substrate. Chamber XXVIII operationalizes the abstract τ-energy through explicit admissibility filters and penalty-based mismatch, thereby instantiating $\mathcal{E}_\tau$ as a measurable survivability budget. Chamber XIV probes the descriptor-level consequences of this budget by exposing a preferred scale $\mu^\star \approx \phi$ as the unique minimizer of a scale-mismatch functional. Together, these chambers do not demonstrate regularity in the classical analytic sense; rather, they empirically test the core UNNS claim that structural persistence and universality arise from collapse-driven selection prior to geometry, smoothness, or PDE evolution.

15 Local Quadratic Stability Near the Scale Attractor

We formalize the observed stability of the scale parameter $\mu^\star \approx \phi$ in Chamber XIV as a local quadratic bound on the scale-mismatch functional.

[Local convexity of scale mismatch] Assume $\Delta_{\text{scale}} : \mathbb{R}_{>0} \to \mathbb{R}_{\geq 0}$ is twice continuously differentiable in a neighborhood of $\mu^\star$ and satisfies

$$\Delta'_{\text{scale}}(\mu^\star) = 0, \qquad \Delta''_{\text{scale}}(\mu^\star) = \lambda > 0.$$

Then for μ sufficiently close to $\mu^\star$,

$$\Delta_{\text{scale}}(\mu) = \Delta_{\text{scale}}(\mu^\star) + \frac{\lambda}{2}(\mu - \mu^\star)^2 + o\Big((\mu - \mu^\star)^2\Big).$$

15.1 Induced Descriptor Update and Local Contraction

Let σ denote the collapse-induced scale selector, implemented in practice by a local descent or minimization step. Assume the update takes the generic form

$$\mu_{k+1} = \mu_k - \eta\, \Delta'_{\text{scale}}(\mu_k),$$

for some effective step parameter $\eta > 0$ determined by the chamber dynamics.

Linearizing near $\mu^\star$ yields

$$\mu_{k+1} - \mu^\star = (1 - \eta\, \lambda)\, (\mu_k - \mu^\star) + o(|\mu_k - \mu^\star|).$$

15.2 Local Contractivity Estimate

Define the local contraction constant

$$L_C := |1 - \eta\, \lambda|.$$

If

$$0 < \eta\,\lambda < 2,$$

then $L_C < 1$ and $\mu^\star$ is a locally attractive fixed point of σ. In particular,

$$|\sigma(\mu) - \mu^\star| \le L_C\,|\mu - \mu^\star| \quad \text{for } \mu \text{ in a neighborhood of } \mu^\star.$$

Interpretation

The convex minimum observed in Chamber XIV is therefore equivalent to the existence of a locally contractive regime of collapse in scale-descriptor space. The golden ratio ϕ appears not as a numerological artifact, but as a descriptor-space attractor selected by τ-admissibility under collapse.

15.3 Numerical Curvature Estimate and an Explicit Local L_C

In chamber practice, $\Delta''_{\text{scale}}(\mu^\star)$ is obtained numerically. Let $\widehat{\mu}^\star$ denote the measured minimizer on a grid and choose a small step $h > 0$ (within the convex neighborhood). Define the central-difference curvature estimate

$$\widehat{\lambda} := \frac{\Delta_{\text{scale}}(\widehat{\mu}^\star + h) - 2\,\Delta_{\text{scale}}(\widehat{\mu}^\star) + \Delta_{\text{scale}}(\widehat{\mu}^\star - h)}{h^2}.$$

Assuming $\widehat{\lambda} > 0$, the same local linearization yields the empirical contraction estimate

$$\widehat{L}_C \;:=\; \left|1 - \eta\,\widehat{\lambda}\right|.$$

Thus, in the scale-descriptor slice, local contractivity is certified by the numerically checkable condition

$$0 < \eta\,\widehat{\lambda} < 2 \quad \Longrightarrow \quad \widehat{L}_C < 1.$$

Equivalently, for μ sufficiently close to $\widehat{\mu}^\star$,

$$|\sigma(\mu) - \widehat{\mu}^\star| \;\le\; \widehat{L}_C\,|\mu - \widehat{\mu}^\star|.$$

Practical note

If the chamber does not expose an explicit step parameter η, one may interpret η as an *effective* local step extracted from the observed update:

$$\widehat{\eta} \;\approx\; \frac{\mu_k - \mu_{k+1}}{\Delta'_{\text{scale}}(\mu_k)} \qquad (\mu_k \text{ near } \widehat{\mu}^\star),$$

and then report $\widehat{L}_C = |1 - \widehat{\eta}\,\widehat{\lambda}|$.

Remark (Downstream only): comparison to RG fixed points

Once a representation layer interprets descriptor updates as a scale transformation on effective models, the locally attractive descriptor $\mu^\star$ plays a role analogous to a renormalization-group fixed point: nearby scale choices flow back toward a canonical value. In UNNS this comparison is strictly downstream: the substrate statement is simply that collapse induces a locally contractive map on descriptors, and any RG-style reading belongs only to later interpretations where geometry and effective field descriptions have already been introduced.

16 Operational Closure and Robustness Criteria

This section completes the formal framework by specifying operational choices required for chamber implementation and by defining robustness conditions that separate genuine emergence from parameter tuning.

16.1 Weight Sequences for τ-Energy

The weight sequence (w_n) encodes how admissibility violations at different nesting depths contribute to $\mathcal{E}_\tau$. The substrate permits several canonical families; the choice must be declared explicitly by any chamber.

- **Finite-horizon weights:**
$$w_n = \begin{cases} 1/N, & n < N, \\ 0, & n \geq N, \end{cases}$$
appropriate when chamber depth is hard-capped at N.

- **Exponential weights:**
$$w_n = (1-\rho)\rho^n, \qquad 0 < \rho < 1,$$
emphasizing near-surface admissibility with controlled memory length.

- **Power-law weights:**
$$w_n = \frac{c}{(n+1)^p}, \qquad p > 1,$$
retaining sensitivity to deep nesting structure.

In all cases, $\sum_n w_n < \infty$ holds by construction. Chambers must log the chosen family and parameters.

16.2 Calibration of the Critical Threshold τ_{crit}

The critical threshold τ_{crit} is not universal; it is calibrated operationally. Three admissible strategies are distinguished:

- **Quantile calibration:**
$$\tau_{\text{crit}} := Q_q(\mathcal{E}_\tau),$$
where Q_q is a high percentile ($q \in [0.90, 0.99]$) of a reference ensemble.

- **Margin calibration:** τ_{crit} is chosen so that $\mathcal{E}_\tau(S) < \tau_{\text{crit}}$ implies all admissibility constraints are satisfied with fixed margin.

- **Phase-separation calibration:** τ_{crit} is selected at the boundary separating collapse-dominated and persistence-dominated regimes.

Chambers must record which calibration strategy is used and the reference data supporting it.

16.3 Multi-Dimensional Descriptor Stability

Let the descriptor space $\mathcal{U} \subset \mathbb{R}^d$ carry a norm-induced metric, and let $\sigma : \mathcal{U} \to \mathcal{U}$ denote the collapse-induced descriptor map. Near a fixed descriptor $u^\star$,

$$\sigma(u) \approx \sigma(u^\star) + J_\sigma(u^\star)(u - u^\star),$$

where J_σ is the Jacobian matrix.

The local contraction constant is

$$L_C^{\mathrm{loc}}(u^\star) := \|J_\sigma(u^\star)\|.$$

A locally stable basin exists if $L_C^{\mathrm{loc}}(u^\star) < 1$. Chambers may estimate J_σ numerically via finite differences and must report L_C^{loc}.

16.4 Inter-Operator Coupling and Basin Persistence

When multiple operators contribute to descriptor evolution, the effective map is a composition

$$\sigma_{\mathrm{tot}} = \sigma_{XII} \circ \sigma_{XV} \circ \sigma_{XIV} \circ \cdots$$

If each component is locally Lipschitz with constants L_i, then

$$L_{\mathrm{tot}} \leq \prod_i L_i.$$

A contractive basin persists if $L_{\mathrm{tot}} < 1$. Dispersive coupling (e.g. from Operator XV) may therefore coexist with local collapse-driven stability provided this inequality holds.

16.5 Emergence versus Tuning

Let p denote the vector of controllable parameters (grid size, depth, λ, σ, etc.), and let $\mu^\star(p)$ denote the observed scale attractor.

Definition 10 (Emergent Attractor)**.** *A descriptor $\mu^\star$ is emergent if, for sufficiently small parameter perturbations δp,*

$$|\mu^\star(p + \delta p) - \mu^\star(p)| \leq K\|\delta p\|,$$

and the local contraction condition $L_C^{\mathrm{loc}} < 1$ persists.

If $\mu^\star$ varies discontinuously or the contractive basin disappears under small perturbations, the value is classified as tuned rather than emergent.

Interpretation

These criteria transform qualitative claims of universality or emergence into testable, chamber-level statements. Collapse universality is thereby grounded not only in formal axioms but in explicit operational protocols.